

# Stochastic Traffic Flow Models of the US National Airspace System

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P. K. Menon<sup>a,1</sup>, M. D. Tandale<sup>a</sup>, J. Kim<sup>a</sup>, P. Sengupta<sup>a</sup>, J. Rosenberger<sup>b</sup>, K. Subbarao<sup>c</sup>

<sup>a</sup>Optimal Synthesis Inc, Los Altos, CA 94022, USA

<sup>b</sup>Department of Industrial and Manufacturing Systems Engineering, The University of Texas at Arlington, Arlington, TX 76019, USA

<sup>c</sup>Mechanical and Aerospace Engineering Department, The University of Texas at Arlington, Arlington, TX 76019, USA

**Abstract:** Stochastic traffic flow models of the US national airspace system based on queuing theory are discussed. Models are developed at multiple spatial resolutions. Specifically, a Center-level model of the airspace is discussed in detail. The model parameters such as the inter-arrival times, service times and traffic flow fractions are derived from the traffic simulation data. The present analysis assumes that the arrivals into the system are Poisson processes, and that the transit times through the airspace segments are described by exponential distributions. The queuing model includes uncertainties introduced by ambient wind, navigation systems and air traffic control advisories. Sample results are given to compare the performance of the queuing model with numerical simulation of the air traffic. Future extensions of the present research are outlined.

Key words: Air Traffic Flow, Queuing Theory, Multiple Resolution, Uncertainty Models

## INTRODUCTION

NASA and the FAA are in the process of transforming the US national air traffic management (ATM) system from airspace-based to trajectory-based operations [1]. Several research initiatives are currently underway within NASA and other leading aviation research centers to help achieve this objective. One of the research goals is the analysis of the impact of trajectory uncertainty and precision on air traffic flow efficiency. Models and simulations of varying fidelity are being developed to realize this goal. At one end of the spectrum are high-fidelity airspace simulation models such as FACET [2] (Future ATM Concepts Evaluation Tool) and ACES [3] (Airspace Concept Evaluation System) which model every aircraft trajectory operating in the airspace together with their performance parameters and flight plans. Analyzing the impact of trajectory uncertainty and precision on the flow efficiency of future traffic concepts using these software packages involves running Monte-Carlo Simulations. The disadvantages of using Monte Carlo simulations are that the results are non-analytic and require enormous amounts of computer time. An alternative

approach is to develop queuing models describing the stochastic influence of the factors affecting the dynamics of the air traffic. This paper describes the development of queuing models at multiple spatial resolutions that can be used to rapidly study the effects various uncertainties and their impact on traffic flow efficiency.

Queuing models are one of the earliest developments in the now well-established field of *Operations Research*. According to Reference 4, much of this theory is attributed to the early works of Erlang [5] in 1917, on the problems in telephony. Although most of applications continued to be in telephony and surface transportation, post WW-II surge in aviation has lead to several applications of the theory to air traffic [6 – 8]. Since then, this modeling methodology has been adopted for addressing various aspects of the air transportation system by the airlines, air cargo fleet operators, and air traffic system designers.

The operating characteristics of queuing systems are determined by two statistical properties, namely, the probability distribution of inter-arrival times and the service times [9, 10]. These distributions can take almost any form in real queuing systems. To be useful, the distributions used in the queuing analysis should be sufficiently realistic, so that the model provides reasonable predictions. At the same time, they should be sufficiently simple so that the model remains tractable. This has prompted the use of exponential distributions in queuing analysis. Queuing models can be

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Email addresses: menon@optisyn.com(P. K. Menon), monish@optisyn.com (M. D. Tandale), jkim@optisyn.com(J. Kim), sengupta@optisyn.com (P. Sengupta), jrosenbe@uta.edu(J. Rosenberger), subbarao@uta.edu(K. Subbarao), URL: <http://www.optisyn.com>.

<sup>1</sup>Corresponding author

characterized by the mean arrival rate  $\lambda$  and the mean service rate  $\mu$ , and can be represented as shown in Figure 1.

Most widely used models in queuing theory are based on the birth-and-death process [9, 10]. Since the mean arrival rate and the mean service rates can be assigned any nonnegative value, these models are said to have a *Poisson* input and exponential service times. Most queuing models differ only in their assumptions about how  $\lambda$  and  $\mu$  change with the number of customers in the queue. In the simplest case, the arrivals are described by *Poisson* processes. The service times are similarly assumed to have exponential distributions.

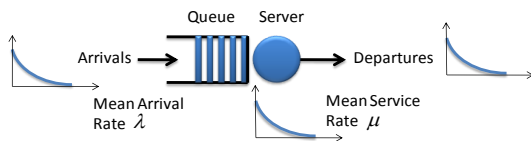


Figure 1. An Elementary Queuing System

More general distribution of inter-arrival times can be modeled using the Erlang's method of serial stages [4, 9] or the more recent method of parallel stages [9]. The resulting queuing models have *Semi-Markovian* [9, 10] properties. While closed-form results are not available for these queues, numerical solutions can be obtained using the Chapman-Kolmogorov equations [9]. This issue will not be pursued any further in this paper.

The next section will discuss the development of air traffic queuing network at a spatial resolution. Models for trajectory uncertainties are outlined in a subsequent section. Additional details about the airspace queuing models and uncertainty models are given in References 11 through 13.

**QUEUING NETWORK MODELS OF THE AIR TRAFFIC SYSTEM**

The US national airspace is organized as a set of Air Route Traffic Control Centers (<http://www.faa.gov/>) covering major population centers of the country. These ARTCCs or Centers are further divided into air traffic control Sectors. Air traffic can be described in terms of their transition through various Centers and Sectors. A more general description can be based on latitude-longitude tessellation of the airspace. Models at all these resolutions have been developed, and are described in detail in Reference 11.

Air traffic flow in the airspace can be formulated as a queuing network. Since any aircraft entering the airspace will eventually leave the system, these are *open* queuing networks. As an example, queuing network of a hypothetical air traffic system with departure airports and four arrival airports is illustrated in Figure 2. As indicated in the previous paragraph, the en-route airspace can be discretized in many different ways. For instance, Sector-level spatial discretization can provide higher resolution than a Center-level discretization.

The service times at each node in the network correspond to the transit time through the corresponding spatial element of the air traffic system. In addition to the arrival rate and service rate distributions for each node, the definition of queuing networks require the specification of flow fractions or routing probabilities  $P_{ij}$  at each branch point. Given the distribution of the traffic entering the system and their flight plans over a specified time interval, a traffic simulation program such as FACET can be used to derive the queuing network parameters.

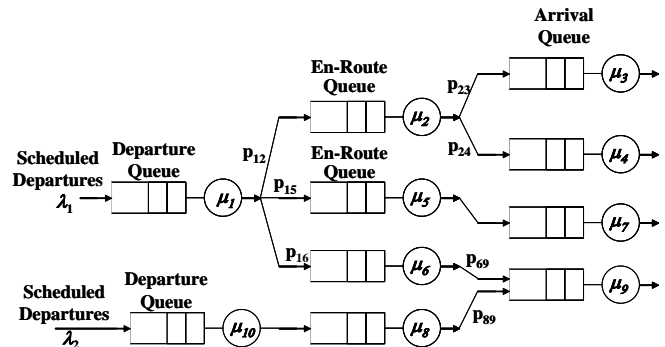


Figure 2. A Sample Queuing Network Representing Two Departure Airports and Four Arrival Airports

A flowchart for deriving the queuing parameters from airspace simulation programs such as FACET is given in Figure 3. The process is based on running the airspace trajectory simulations forward by a fixed time step and analyzing the motion of aircraft between regions defining the spatial discretization of the airspace. For instance, in order to derive the parameters of the Center-level model, movement of aircraft between Air Route Traffic Control Centers will be examined to determine the service time distribution at each Center, and the corresponding flow fractions.

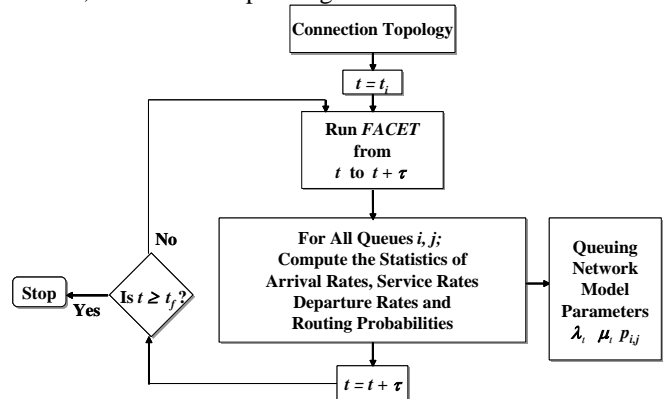


Figure 3. Derivation of Queuing Network Parameters from Trajectory Simulations

Several other air traffic queuing network models of varying fidelity have been suggested in the literature. For instance, Reference 14 used nested queuing models to describe the air traffic interactions with the air traffic control

center. Reference 15 has developed a national airspace system demand and capacity model. Reference 16 has developed a queuing model for analysing aviation policies based on air traffic delays. The Logistics Management Institute (LMI) developed a queuing network model of the NAS called *LMINET* [17]. Several other airspace queuing models are discussed References 18 through 23.

Although several air traffic queuing models have been described in the literature, none of them have considered the effects of trajectory uncertainties due to aviation operations and precision of navigation and control on the traffic flow efficiency. The present research seeks to address this issue. Other contributions of the present research are an automatic methodology for deriving the queuing model parameters from traffic simulation data, and the development of queuing models at multiple resolutions.

Since the trajectory uncertainties and precision affect the air traffic system differently at national, regional and local levels, it is often necessary to develop multi-resolution queuing models. For instance, national level queuing network model of the Class-A airspace can be built in terms of the topology of the jet-routes (<http://www.faa.gov/>). Although most of the traffic in the current air traffic system tends to follow the jet-routes, more advanced en route procedures such as Direct-to [24] can cause aircraft to deviate from these routes, introducing inaccuracies. A more flexible queuing network model of the airspace can be constructed by partitioning the airspace using a latitude-longitude tessellation. Following the previous work on aggregate traffic flow modeling [25 – 28], each tessellation can be assumed to be *8-connected*. Queuing network can then be defined in terms of this topology.

Both these networks will contain several queues each involving service time distributions and traffic flow fractions. Such detail may not be desirable in certain studies. In those cases, a more compact queuing model can be constructed by adopting the Air Route Traffic Control Center-level network topology advanced in References 25 through 32. For the sake of clarity, a schematic diagram illustrating the connectivity between various nodes of this network is given in Figure 4.

This network illustrates the connections between the 20 Air Traffic Control Centers in the continental United States. Unlike the topologies employing latitude-longitude tessellation of the airspace, since the dimensions of the Centers are not uniform, the service time distributions in this queuing network cannot be explicitly related to the geometry of the airspace.

The discussions in the present paper will be focused on the Center-level model of the US national airspace system.

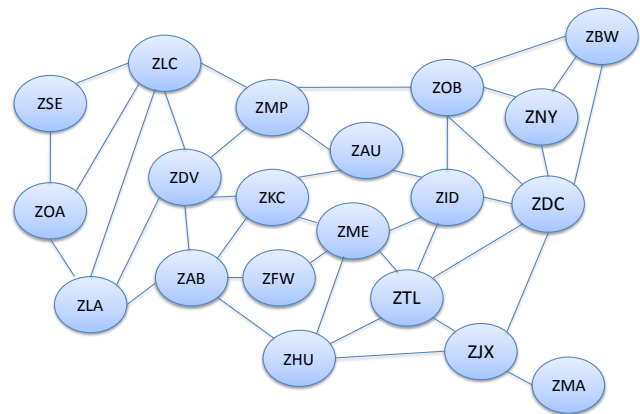


Figure 4. Topology for the Center-Level Queuing Network Model of the NAS

### MODELS FOR TRAJECTORY UNCERTAINTIES

The objective of this research discussed in this paper is to analyze the impact of trajectory uncertainty and precision on the traffic flow efficiency using queuing theory as the modeling tool. The approach models several quantifiable uncertainties in the air transportation system. These uncertainties, together with their impact on the aircraft trajectories are illustrated in Figure 5.

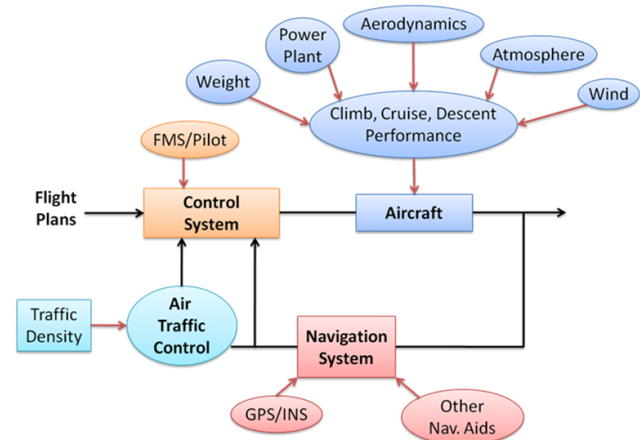


Figure 5. Models for Uncertainties in Aviation Operations and Precision of Navigation and Control

Weight, power plant, aerodynamics, and atmospheric variations introduce in climb and descent times. En-route winds and air traffic advisories will cause variations in time of flight between regions of the airspace while in cruise. Additional trajectory uncertainties may be introduced by the navigation systems and aircraft pilotage [33].

The uncertainties in aviation operations and the precision of navigation and control can be expressed in terms of uncertainties the position and velocity vectors of aircraft. These can then be transformed into service time distributions of the airspace in the queuing network.

For instance, an uncertainty model can be formulated based on the observation that the sensitivities of time-to-climb and time-to-descend are approximately linear with respect to aircraft parameters. The BADA model for aircraft climb and descent [34] is given by a first-order differential equation in terms of thrust  $T$ , Drag  $D$ , Airspeed  $V$ , Weight  $W$ , and the climb-descent schedule  $f(M)$  as:

$$\frac{dh}{dt} = \frac{(T-D)V}{W} f(M) \quad (1)$$

Here,  $M$  is the Mach number.

Stochastic integration of the inverse of the differential equation (1) can be used to derive the statistics of time-to-climb and time-to-descend. In the general case, this can only be accomplished through a Monte-Carlo simulation.

However, based on the approximate linearity of the sensitivities, the inverse of equation (1) can be linearized using Taylor series expansion of the right hand side as:

$$\frac{dt}{dh} = F(x) \approx F(x_0) + \nabla F|_0 \cdot (x - x_0) \quad (2)$$

$$x = [W \quad V \quad T \quad D \quad \rho]^T$$

Variations in aircraft weight, airspeed, thrust, drag and air density are assumed in the following development. The perturbation term  $\nabla F \cdot \delta x$ , can be expanded in Taylor series to yield:

$$\begin{aligned} \nabla F|_0 \cdot \delta x &= \frac{\partial F}{\partial W}|_0 \delta W + \frac{\partial F}{\partial V}|_0 \delta V + \frac{\partial F}{\partial T}|_0 \delta T \\ &+ \frac{\partial F}{\partial D}|_0 \delta D + \frac{\partial F}{\partial \rho}|_0 \delta \rho \end{aligned} \quad (3)$$

The partial derivatives on the right hand side can be evaluated along the nominal climb-descent path specified by the BADA model. In Equation (3),  $\delta W$  is the aircraft weight uncertainty at takeoff or top of descent,  $\delta T$  is the power plant uncertainty,  $\delta D$  is the drag uncertainty due to the takeoff lift coefficient uncertainty and  $\delta \rho$  is the uncertainty in atmospheric density.

If the uncertainties in aircraft parameters are assumed to be distributed as zero-mean Gaussian processes, Equation (3) can be integrated to yield the variance in climb-descent time as:

$$\begin{aligned} \text{Var}(t_{climb}) &= \sigma_W^2 \left( \int_{h_1}^{h_2} \frac{\partial F}{\partial W} dh \right)^2 + \sigma_V^2 \left( \int_{h_1}^{h_2} \frac{\partial F}{\partial V} dh \right)^2 \\ &+ \sigma_T^2 \left( \int_{h_1}^{h_2} \frac{\partial F}{\partial T} dh \right)^2 + \sigma_D^2 \left( \int_{h_1}^{h_2} \frac{\partial F}{\partial D} dh \right)^2 \\ &+ \sigma_\rho^2 \left( \int_{h_1}^{h_2} \frac{\partial F}{\partial \rho} dh \right)^2 \end{aligned} \quad (4)$$

Here,  $\sigma_{(\cdot)}$  is the standard deviation of each uncertainty component. Note the integrands in Equation (4) depend on the type of aircraft. Consequently, the fleet mix must be specified in order to enable the derivation of the service time distributions for use with the queuing network models.

As an example, Figure 6 illustrates the climb time distribution for equal mix of three aircraft types, with 10% standard deviations in takeoff weight uncertainties.

Similar arguments can be employed to model the trajectory uncertainties from other sources listed in Figure 5. In the interests of brevity, these models will not be discussed here. Detailed accounts of these models are given in References 11 and 13.

The service time variations due to trajectory uncertainties can be combined with nominal service time distributions to yield the total service time distributions at each node. In the present research, nodes incorporating the service time uncertainties are appended in cascade with the nominal service nodes to capture the impact of uncertainties on the traffic flow.

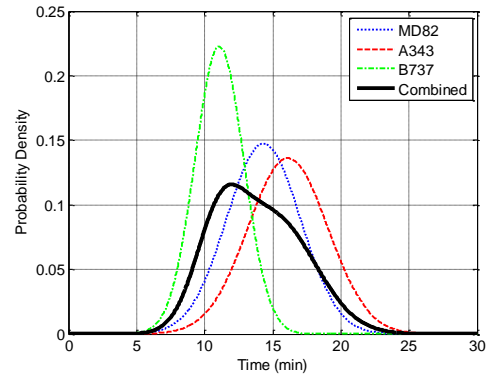


Figure 6. Probability distribution of the time-to-climb from 0 to 32000 (ft) due to 10% 1- $\sigma$  Weight Variations for the Mixed Aircraft Fleet of MD82, A343 and B737

## CENTER-LEVEL QUEUING MODEL OF THE US AIRSPACE

As discussed in one of the previous sections, the key parameters in a Center-level queuing network model are:

- 1) Inter-arrival time distributions at the airports
- 2) Service time distributions in each Center
- 3) Service time uncertainty distributions due to trajectory uncertainties
- 4) Flow fractions between Centers

All except the third item for a Center-level queuing network model was obtained by running the FACET simulation in conjunction with the methodology described in the flowchart given in Figure 3. The uncertainty models in Item 3 were derived using approximate analytical models such as the one described by Equation (4). The inter-arrival time, service time and flow fractions were collected over the simulation propagation horizon, 24 hours in this case.

Statistical distributions were then fitted for use in the queuing analysis.

Traditionally, exponential distributions are used in queuing network analysis primarily because closed-form results are available for these queues. The exponential distribution ( $k=1$ ) captures the inter-arrival time distributions rather well in the airspace under consideration, as can be observed from Figure 7. That figure also shows an additional fit in terms of a second-order Erlang distribution ( $k=2$ ).

Although it is desirable to use exponential distributions for service time distributions as well, it is clear from Figure 8 that the service time distributions are closer to second-order or third-order Erlang distributions. Since closed-form results are not available for queuing networks with these service disciplines, all the results reported in this paper were generated using exponential service time distributions. It will be seen subsequently that the results are quite good even under this approximation. Work is presently underway to employ more recent queuing approximations [35] that allow better representations of the service time distributions.

The third data elements necessary for constructing the queuing network are the flow fractions between the Centers. As in the case of inter-arrival time distributions and the service time distributions, these data components can be derived as a part of the computations outlined in the flowchart given in Figure 3. As an example, Figure 9 illustrates the flow fractions from the Chicago Center to neighboring Centers at 20-minute intervals. It may be observed that the flow fractions are more or less constant over the 12-hour period.

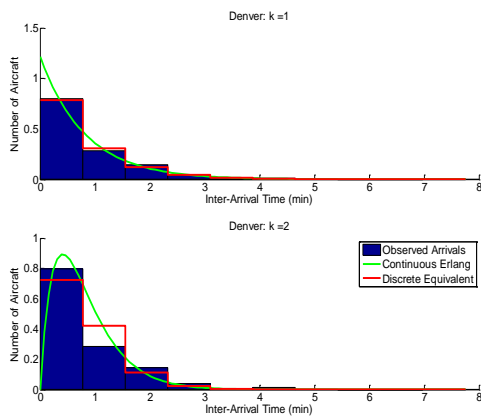


Figure 7. Inter-Arrival Time Distributions for the Denver Center (ZDV)

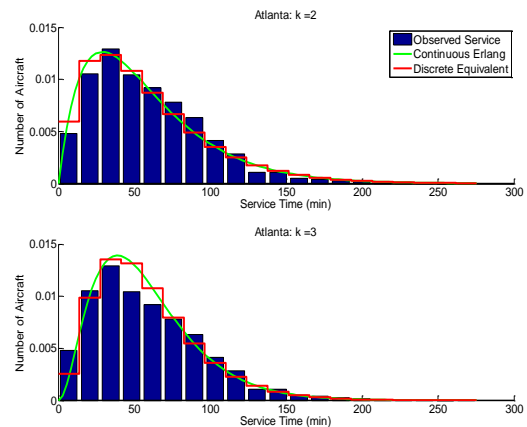


Figure 8. Service Time Distributions for the Atlanta Center (ZTL)

These distributions can be used to set up the Center-level queuing network. Queuing models at other spatial resolutions can be derived in an entirely analogous manner. However, these will not be discussed in the present work. Interested readers may find additional details in References 11 and 12.

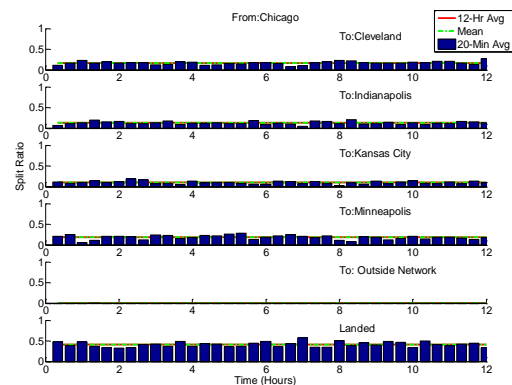


Figure 9. Flow Fractions for Flights leaving the Chicago Center

### Analysis of the Center-Level Network

Under the assumptions of exponential inter-arrival and service time distributions, and constant flow fractions, the Center-level queuing model is a Jackson network [9]. Jackson networks can be characterized as a network of  $N$  service nodes where each service node  $j$  ( $j = 1..N$ ) has an infinite waiting space in the queue.

1. Aircraft arrive into the Center from outside the system according to a Poisson input process (Exponential Inter-Arrival Times) with mean arrival rate  $a_j$ .

2. Each Center or node in the network has  $m_j$  parallel servers with exponential service time distribution with mean service rates  $\mu_j$ .
3. Aircraft leaving the Center or node  $i$  is routed to an adjacent node  $j$  with probability  $p_{i,j}$  or departs the system with probability  $q_j = 1 - \sum_{i=1}^N p_{i,j}$ .

The previous section illustrated sample inter-arrival time distributions, service time distributions and flow fractions for the Center-level Jackson network. The number of parallel servers in each Center was determined by adding the capacities of all the Sectors in the center.

Under steady state conditions, each node  $j$  in the Jackson network can be treated as if it were an independent  $M/M/m_j$  queuing system [9] with arrival rate  $\lambda_j$  obeying the flow-balance equation

$$\lambda_j = a_j + \sum_{i=1}^N \lambda_i p_{i,j} \quad (5)$$

Here,  $m_j \mu_j > \lambda_j$  to ensure that the queues at every node can achieve steady-state. The flow-balance equation can be solved as:

$$\lambda = (I - p^T)^{-1} a^T \quad (6)$$

After calculating the arrival rate  $\lambda$ , each node is analyzed independently using the following formulae. Note that the formulae used in the analysis can be found in most textbooks on queuing theory. The formulae for the present analysis are from References 9 and 10.

Let  $P_{nj}$  indicate the probability that  $n$  customers are present at node  $j$ . The quantities  $P_{0j}$  and  $P_{nj}$  are calculated as:

$$P_{0j} = \frac{1}{\sum_{i=0}^{m_j-1} \frac{(\lambda_j / \mu_j)^i}{i!} + \frac{(\lambda_j / \mu_j)^{m_j}}{m_j!} \frac{1}{1 - \lambda_j / (m_j \mu_j)}} \quad (7)$$

$$P_{nj} = \begin{cases} \frac{(\lambda_j / \mu_j)^n}{n!} P_{0j} & \text{if } 0 \leq n < m_j \\ \frac{(\lambda_j / \mu_j)^n}{m_j! m_j^{(n-m_j)}} P_{0j} & \text{if } n \geq m_j \end{cases} \quad (8)$$

Expected queue length at node  $j$  (excluding aircraft being served in each Center) is given by:

$$L_{qj} = \frac{P_{0j} (\lambda_j / \mu_j)^{m_j} \rho_j}{m_j! (1 - \rho_j)^2} \quad (9)$$

with  $\rho_j = \lambda_j / (m_j \mu_j)$ .

Expected number of the aircraft at the node (both being served and waiting) is given by:

$$L_j = L_{qj} + \frac{\lambda_j}{\mu_j} \quad (10)$$

Expected waiting time in queue (excluding the time for service) is:

$$W_{qj} = \frac{L_{qj}}{\lambda_j} \quad (11)$$

Expected system time (including both waiting and service times) is:

$$W_j = W_{qj} + \frac{1}{\mu_j} \quad (12)$$

Equations (7) through (12) will next be used to generate the results from the Center-level queuing model and comparisons with actual air traffic simulation (FACET).

Figure 10 illustrates the probability distribution of the number of aircraft  $P_n$  in the Seattle Center as predicted by the queuing network and compares it with the mean number of aircraft in the Seattle Center observed over a 12-hour FACET simulation. It may be observed that the mean of the probability distribution computed using the queuing model matches closely with the mean from the FACET traffic simulation. Similar results have been observed at all other Centers.

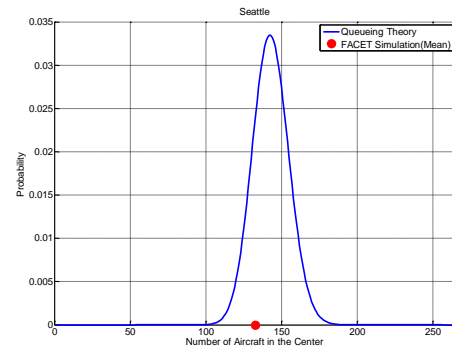


Figure 10. Number of Aircraft in the Seattle Center

Figure 11 shows the comparison between the mean numbers of aircraft in each Center as observed in the FACET simulation and as calculated by the queuing network model. The results show that the queuing model slightly overestimates the mean number in the Centers.

Figure 12 shows the inter-departure time distribution for network outputs at airports as observed in the FACET simulation and as calculated from the queuing model for the Denver Center. Good match between the two can be observed. Figure 13 shows the mean network output rates at airports in aircraft/ minute. As in the case of the mean number of aircraft in the Center, the queuing model tends to over estimate the output rate at the airports when compared with the FACET simulation.

As mentioned earlier, the number of parallel servers  $m_j$  for each Center was determined by summing the Sector capacities of all Sectors within that Center. This yields an unrealistically large number of parallel servers for each Center, leading to low traffic flow intensity ( $\rho = \lambda / m\mu$ ). One of the consequences of such an aggregation is the loss of spatial resolution since the en-route Sectors may be sparsely occupied while the terminal area Sectors may be saturated. Although the Center capacity may not be exceeded, delays may be introduced due to saturated terminal area Sectors. This effect is not captured by the present Center-level model and finer spatial resolutions such as Sector-level models or latitude-longitude tessellation based models may be required to provide better representation of traffic. This approach will increase the dimension of the network with attendant complexities in the interpretation of the results. The chief advantage of the Center-level model is its lower dimension, yielding a more macroscopic view of the national traffic flow.

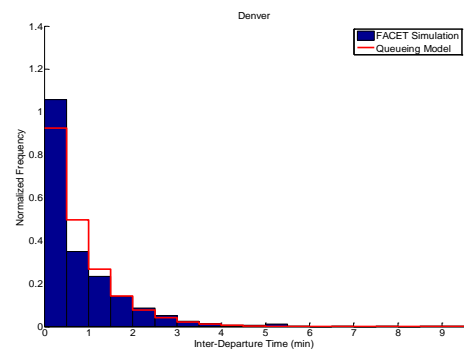


Figure 12. Inter-Departure Time Distribution at the Network Outputs (Landing Flights)

### CONCLUSIONS

This paper discussed the development of stochastic queuing network models of the US airspace system based on spatial discretization of the airspace. The Center-level model was discussed in additional detail. Derivation of the queuing network parameters from an air traffic simulation system was discussed, together with the approximations necessary for deriving analytical results. Approximation of the inter-arrival time and service time distributions by exponential distributions allowed the treatment of the Center-level queuing model as a Jackson network. Sample results from the queuing network model compare favourably with those from explicit numerical simulation of the air traffic, even under the restrictive assumptions employed in this work.

Additional queuing network models of the national airspace system are currently under development. Validations efforts using Monte-Carlo simulations are also underway.

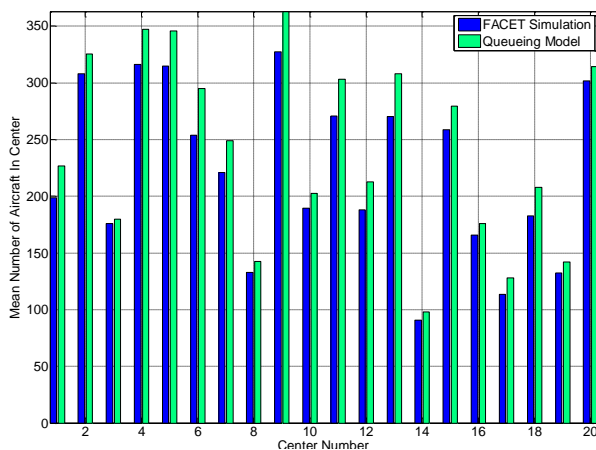


Figure 11. Mean Number of Aircraft at various Centers predicted by trajectory simulations and the queuing network

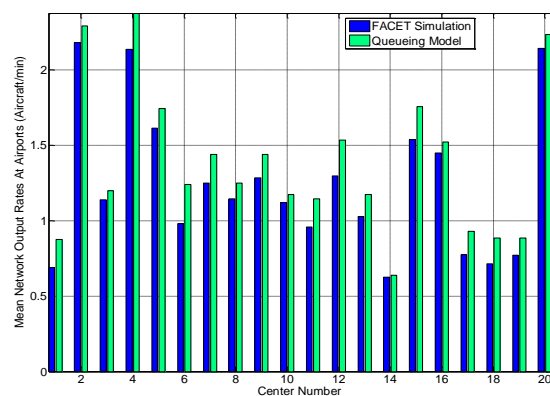


Figure 13. Mean Network Output Rates at Airports

### ACKNOWLEDGEMENT

This research is supported under NASA Contract No. NNA07BC55C, with Mr. Michael Bloem and Ms. Jane Thipphavong as the Technical Monitors. Ms. Rebecca Grus

served as the Contracting Officer's Technical Representative.

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**P. K. Menon** is the President and Chief Scientist of Optimal Synthesis Inc., an aerospace systems technology company focusing on research and development. He was previously with the Georgia Institute of Technology, NASA Ames Research Center, Integrated Systems, Inc and ISRO. He has also been an adjunct faculty at the Santa Clara University. Dr. Menon received a Ph. D degree from Virginia Tech (1983), a M.E. in Aeronautical Engineering from IISc (1975), and a B. E. in Mechanical Engineering from Regional Engineering College, Warangal (1973). His current interests are Air Transportation Systems and Nonlinear Estimation Theory.



**Monish D. Tandale** is a Research Scientist at Optimal Synthesis Inc. He received his Ph. D. and master's degrees in Aerospace Engineering from Texas A&M University in 2006 and 2002 respectively, and a bachelor's degree in Mechanical Engineering from the University of Mumbai in 2000. His current interests are Air Transportation Systems and High-Performance Computing Systems.



**Jinwhan Kim** is a Research Scientist at Optimal Synthesis Inc. He received his Ph. D. and master's degrees in Aeronautics &

Astronautics from Stanford University in 2007 and 2002, masters and bachelor's degrees in Naval Architecture and Ocean Engineering from Seoul National University in 1995 and 1993. His current interests are Nonlinear Estimation Theory and Robotics.



**Prasenjit Sengupta** is a Research Scientist at Optimal Synthesis Inc. He received his Ph. D. and master's degrees in Aerospace Engineering from Texas A&M University in 2006 and 2003 respectively. He received his bachelor's degree in Aerospace Engineering from IIT, Kharagpur in 2001. His current interests are Air Transportation Systems, and Optimal and Nonlinear Dynamic Systems.



**Jay Rosenberger** is an Assistant Professor in the Department of Industrial & Manufacturing Systems Engineering at The University of Texas-Arlington. He received his Ph.D. in Industrial Engineering from Georgia Institute of Technology in 2001, and M.S. in Operations Research from University of California-Berkeley in 1997. His research interests include the applications of Mathematical Programming, Stochastic Optimization, and Simulation.



**Kamesh Subbarao** is an Assistant Professor in the Department of Mechanical & Aerospace Engineering at The University of Texas-Arlington. He received a Ph.D. from Texas A&M University in 2001 and M.E. from Indian Institute of Science in 1995. His research interests include Dynamical Systems Theory, Multi-Resolution Mathematical Modeling and Simulation methodologies.