

Queuing Network Models of the National Airspace System

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Understanding the relationships between trajectory uncertainties due to aviation operations, precision of navigation and control, and the traffic flow efficiency are central to the design of next generation Air Transportation Systems. Monte-Carlo simulations using air traffic simulation software packages can be used to quantify these effects. However, they are generally time consuming, and do not provide explicit relationships for comparing various technology options. On the other hand, queuing models of the air traffic system can rapidly demonstrate the influence of trajectory uncertainties on traffic flow efficiency, facilitating tradeoff studies in an effective and time-efficient manner. A methodology for incorporating the trajectory uncertainty models into queuing network models of the air traffic at national, regional and local scales is discussed. Usefulness of these models in assessing the impact of uncertainties on traffic flow efficiency is illustrated.

I. Introduction

NASA and the FAA are in the process of transforming the national air traffic management (ATM) system from airspace-based to trajectory-based operations¹. The current air traffic management methodology is based on a fixed airspace structure tied to geographic locations within the National Airspace System (NAS), and can be termed as Fixed Airspace Operations. The Trajectory-Based Operations is a paradigm shift from the current approach and uses four-dimensional (4-D) trajectories as the basis for managing the ATM system. In Trajectory-based operations (TBO), all ATM decisions across all time horizons, are fundamentally related to 4-D trajectories. Several research initiatives are currently underway within NASA to help achieve this transformation from airspace-based to trajectory-based operations.

One of the research goals is the analysis of the impact of trajectory uncertainty and precision on air traffic flow efficiency. Models and simulations of varying fidelity are being developed to realize this goal. At one end of the spectrum are high-fidelity airspace simulation models such as FACET² (Future ATM Concepts Evaluation Tool) and ACES³ (Airspace Concept Evaluation System) which model each aircraft together with its performance parameters and flight plan. Analyzing the impact of trajectory uncertainty and precision on the flow efficiency of future traffic concepts using these software packages involves running Monte-Carlo simulations. The disadvantages of using Monte Carlo simulations are that the results are non-analytic and may require enormous amounts of computer time. An alternative approach is to develop queuing models describing the stochastic influence of the factors affecting the air traffic dynamics.

Queuing models are one of the earliest developments in the now well-established field of *Operations Research*. According to Reference 4, much of this theory is attributed to the early works of Erlang⁵ in 1917, on the problems in telephony. Although most of applications continued to be in telephony and surface transportation, the post WW-II surge in aviation has lead to several applications of the theory to air traffic⁶⁻⁸. Since then, this modeling methodology

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has been adopted for addressing various aspects of the air transportation system by the airlines, air cargo fleet operators, and air traffic system designers.

Several air traffic queuing network models have been suggested in the recent literature. For instance, Modi⁹ used nested queuing models to describe how the air traffic interacts with the air traffic control center. Polhemus¹⁰ compared continuous autoregressive models with traditional queuing models. Frolow and Sinnott¹¹ developed the National Airspace System Performance Analysis Capability (NASPAC) to model the NAS. Wieland¹² implemented the Detailed Policy Assessment Tool (DPAT) to analyze delays in the NAS. The Logistics Management Institute (LMI) developed a queuing network model of the NAS called *LMINET*¹³. More recently, Callaham and Wieland¹⁴ studied the existence of chaotic behavior in network queuing models of the NAS. Rakas and Schonfeld¹⁵ used deterministic queues to analyze the effects of degraded capacity on the NAS. Bäuerle et al.¹⁶ considered the affects of turbulence on inbound traffic using queuing models. Wieland¹⁷ tested the hypothesis whether air traffic can be modeled using queuing systems. Cavcar and Cavcar¹⁸ studied the impact of aircraft performance for departing aircraft on air traffic delays. Shortle and Mark¹⁹ presented methods for reducing the complexity of airspace queuing networks namely, removal of low-utilization queues and clustering a group of nodes in to a single node. However, they used the queuing networks for efficient simulation of the NAS and not for carrying out any assessment of the effects of uncertainties on air traffic efficiency.

Although several air traffic queuing models have been described in the literature, none of them of them have considered the effects of trajectory uncertainties due to aviation operations and precision of navigation and control on the traffic flow efficiency. This paper describes the development of multiple resolution queuing models that allow the rapid assessment of the relationships between various trajectory uncertainties and the traffic flow efficiency metrics. Section II describes the multi-resolution queuing network modeling of the NAS. The various trajectory uncertainties modeled in this paper are briefly introduced in Section III. Further details about the uncertainty modeling are provided in a companion paper²⁰. Section IV presents some preliminary results. Finally conclusions are presented in Section. VII

II. Queuing Network Models of Air Traffic System

The operating characteristics of queuing systems are largely determined by two statistical properties, namely, the probability distribution of inter-arrival times and the service times^{21, 22}. These distributions can take almost any form in real queuing systems. However, in order to formulate a queuing model as a representation of the real system, it is necessary to specify the assumed form of each of these distributions. To be useful, the assumed form should be sufficiently realistic, so that the model provides reasonable predictions while at the same time being tractable. The desire to maintain analytical tractability has prompted the use of exponential distributions which simplify the solution process. Queuing models are often characterized by the mean arrival rate λ and the mean service rate μ , and are generally represented as shown in Figure 1.

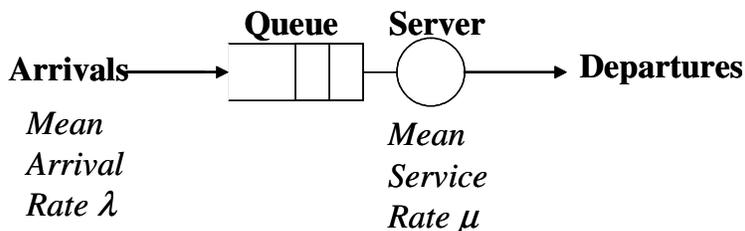


Figure 1. An elementary queuing system

Most widely used models in queuing theory are based on the birth-and-death process^{21,22}. Since the mean arrival rate and the mean service rates can be assigned any nonnegative value, these models are said to have a *Poisson* input and exponential service times. Most queuing models differ only in their assumptions about how λ and μ change with the number of customers in the queue. In the simplest case, the arrivals are described by *Poisson* processes. The service times are similarly assumed to have exponential distributions.

To get a realistic model, more general distributions of inter-arrival times can be modeled using the Erlang's method of serial stages^{4,21} or the recent method of parallel stages²¹ and Coxian queues. The resulting queuing models have the *Semi-Markovian*^{21,22} properties, allowing the application of powerful theoretical results in stochastic systems theory. More recently, Reference 23 demonstrated how additional state variables can be introduced in the queuing system for transforming non-Markovian models to Markovian form. Approaches for handling the non-Markovian nature of the queues using Erlang and Coxian queues can generally be termed as phase-type

distributions²⁴. The main problem here is to find a phase-type distribution, defined as the time to absorption in a discrete-state continuous-time Markov chain that approximates a given distribution in an optimal manner.²⁵⁻²⁷

Air traffic systems are generally formulated as a queuing network. Since any aircraft entering the system will eventually leave the system, these are *open* queuing networks. The queuing network of a hypothetical air traffic system is illustrated in Figure 2. It is important to note that the network topology depends upon the nature of the traffic flow being considered. For instance, the queuing network in Figure 2 is suitable for national level traffic flow studies. Different network topologies will be required for regional and local level traffic flow analysis.

The service times at each node in the network correspond to the transit time through each component of the air traffic system. In addition to the arrival rate and service rate distributions for each node, queuing networks require the definition of routing probabilities P_{ij} at each branch point. Given the distribution of the traffic entering the system and their flight plans over a specified time interval, a traffic simulation program such as FACET can be used to compute the queuing network parameters.

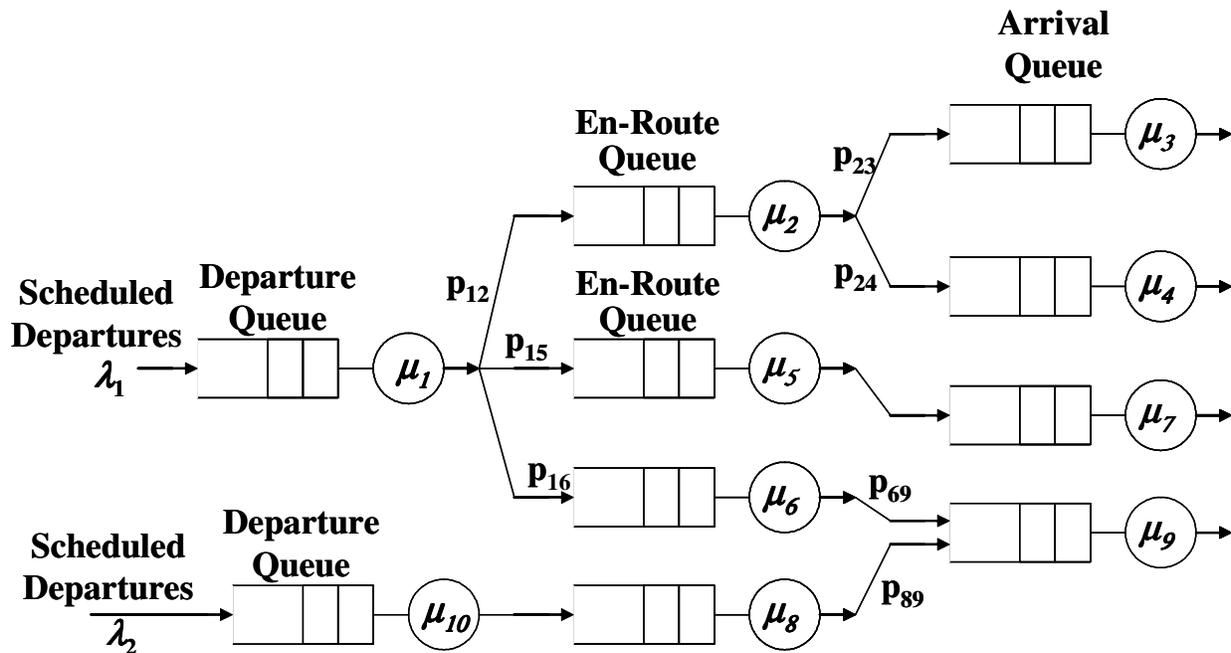


Figure 2. A sample queuing network representing two departure airports and four arrival airports

Trajectory uncertainties and precision affect the air traffic system differently at national, regional and local levels, so multi-resolution queuing models must be developed. For instance, national level queuing network model of the Class-A airspace can be built in terms of jet-route topology shown in Figure 3. Although most of the traffic in the current air traffic system tends to follow the jet-routes, advanced en route procedures such as Direct-to²⁸ can cause aircraft to deviate from these routes, introducing inaccuracies in the model. A more flexible queuing network model of the airspace can be constructed by partitioning the airspace using a latitude-longitude tessellation. Following a previous work on aggregate traffic flow modeling²⁹⁻³², each tessellation can be assumed to be *8-connected*. Queuing network can then be defined in terms of this topology.

Both these networks will contain several queues each involving service time distributions and routing probabilities. However, such detail may not be desirable in certain studies. In those cases, a more compact queuing model can be constructed by adopting the Air RouteTraffic Control Center level network topology advanced in References 33 through 36. For the sake of clarity, this network is given in Figure 4. The network is completely described by the connections between the 20 Air Route Traffic Control Centers. Unlike the other two topologies, the service time distributions in this queuing network cannot be explicitly related to the geometry of the airspace.

In order to enable rapid analysis, an automatic numerical algorithm is developed for assembling queuing network models directly from traffic simulations. Preliminary results on assembling a center-level queuing network model from a FACET playback run are presented in Section IV.

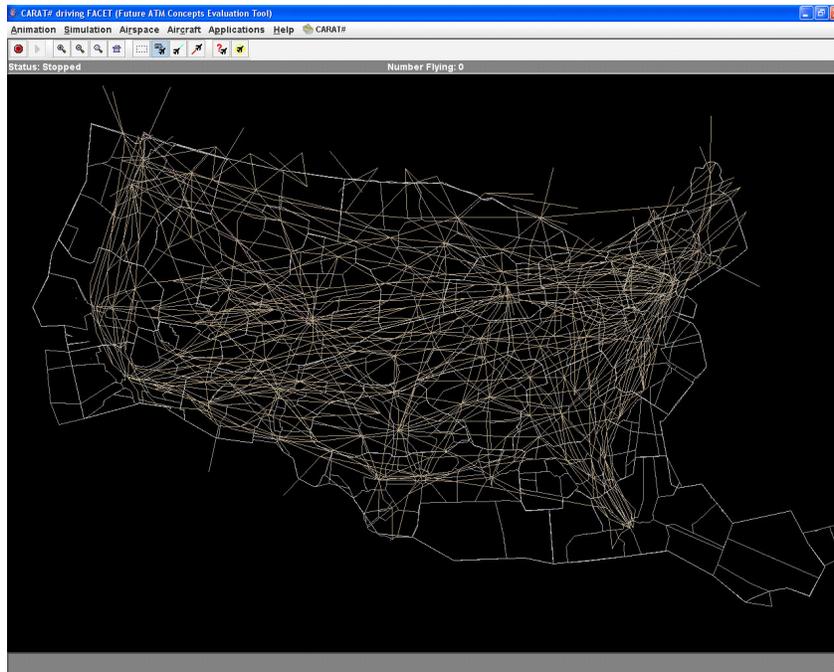


Figure 3. Jet routes over the continental US

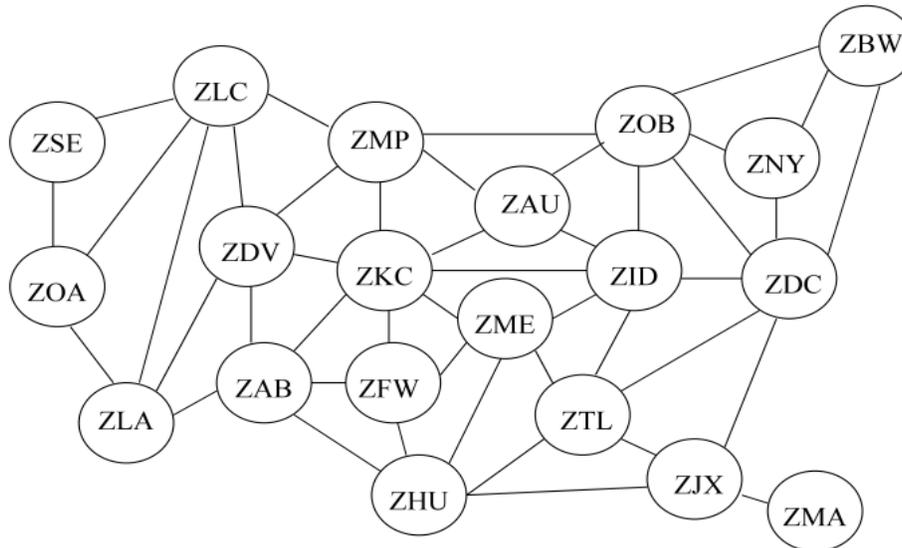


Figure 4. Topology for a center level queuing network model of the NAS³⁴

III. Models for Trajectory Uncertainties

The central objective of the research discussed in this paper is to analyze the impact of trajectory uncertainty and precision on the traffic flow efficiency using Queuing Theory as the modeling tool. The approach models every quantifiable uncertainty in the air transportation system. Details of this modeling effort are described in detail in a companion paper²⁰. Once models capturing the effects of the trajectory uncertainties and precision are available, statistical methods³⁷ can be used to relate them to the queuing network model parameters. The uncertainties in aviation operations and the precision of navigation and control can be expressed in terms of the position and velocity

vectors. These can then be transformed into service rate distributions in the queuing network models. The conceptual approach is illustrated in Figure 5. The queuing network model can then be used to analytically quantify the traffic flow efficiency through the air traffic system.

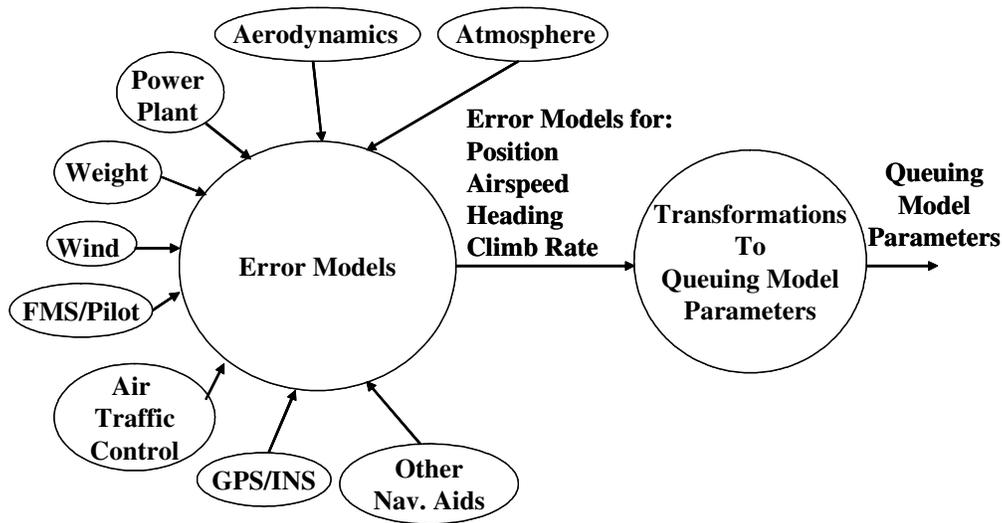


Figure 5. Models for uncertainties in aviation operations and precision of navigation and control

IV. Estimating Queuing Model Parameters from NAS Traffic Data

The key elements necessary for formulating a queuing network model are:

- 1) The inter-arrival time or arrival rate distributions at the edges of the network
- 2) The service time distributions at each node
- 3) Routing probabilities
- 4) Number of parallel servers per node

These network parameters for a center-level queuing network model in Figure 4 can be obtained by running a playback of the NAS flight data in FACET. The inter-arrival time, service time and routing probability can be collected over the playback propagation horizon. The number of parallel servers per node can be estimated by analyzing the maximum number of aircraft that can be simultaneously served by the airspace under consideration.

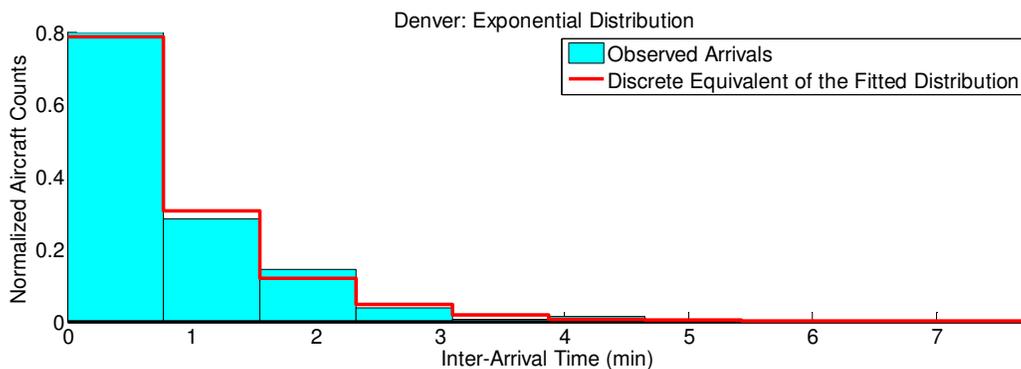


Figure 6. Inter-arrival time distribution fit for the Denver center (ZDV)

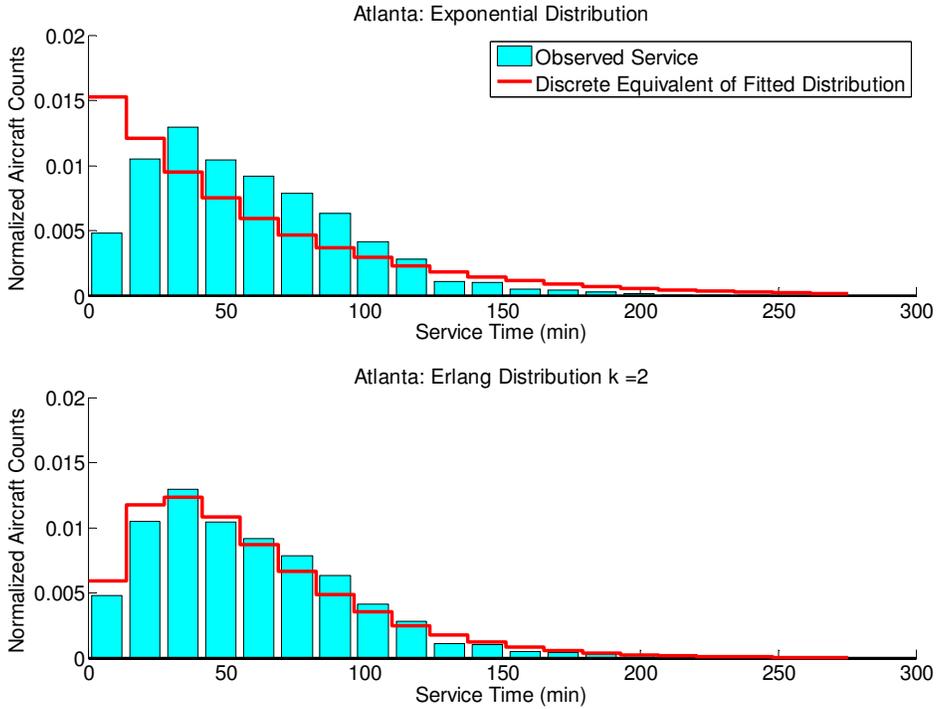


Figure 7. Service Time Distributions Fits for the Atlanta Center (ZTL)

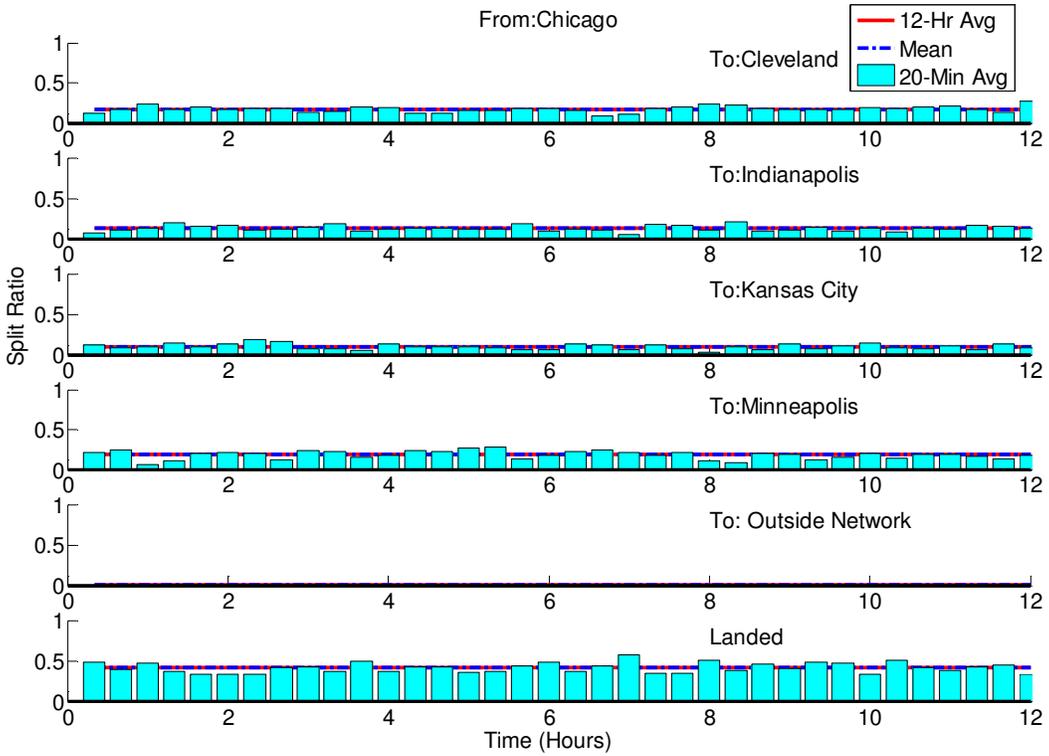


Figure 8. Routing Probabilities for Flights leaving the Chicago Center

Figure 6 through Figure 8 illustrate the inter-arrival time distribution, service time distribution and the transition probabilities observed from a FACET playback run. Distributions generally used in queuing models can be fitted to these observed statistics for further analysis. Traditionally, exponential distributions have been used in queuing model analysis as this allows the derivation of closed-form solutions. The inter-arrival time distribution at the Denver Center obtained from FACET is found to be exponential as shown in Figure 6. However the service time distributions at Atlanta were found to be stage 2 or stage 3 Erlang distributions as shown in Figure 7(b). Nevertheless, the service time will be assumed to be exponentially distributed as in Figure 7(a) to maintain mathematical tractability. Future research will address queuing network analysis with other distributions that fit the service processes more closely. Note that the mathematical tractability of the queuing model is lost when non-exponential distributions are used and closed form expressions for the queuing results described in Section V are not available. Future analysis of queuing networks with non-exponential distributions will resort to approximations or numerical techniques.

Using the key elements identified in this section such as inter-arrival time, service time and transition probabilities, a Center-Level Open Jackson Queuing network with M/M/m nodes can be constructed and analyzed further as described in the following section.

V. Analysis of Center-Level Open Jackson Network Model

A Jackson Network²² can be characterized as a network of N service nodes where each service node j ($j = 1..N$) has an infinite waiting space in the queue.

- 1) Customers arrive from outside the system according to a Poisson input process (Exponential Inter-Arrival Times) with mean arrival rate a_j .
- 2) Each node has m_j parallel servers with exponential service time distribution having mean service rate μ_j .
- 3) A customer leaving node i is routed to an adjacent node j with probability $p_{i,j}$ or departs the system with probability $q_j = 1 - \sum_{i=1}^N p_{i,j}$.

The previous section described the procedure to obtain inter-arrival time distributions, service time distributions and transition probabilities from a FACET playback run, which are used to construct a Center Level Open Jackson Network Model. It is known that under steady state conditions, each node j in the Jackson Network behaves as if it were an independent $M/M/m_j$ queuing system with arrival rate λ_j obeying the flow-balance equation

$$\lambda_j = a_j + \sum_{i=1}^N \lambda_i p_{i,j} \quad (1)$$

where $m_j \mu_j > \lambda_j$ will ensure that the steady-state can be attained. The matrix form of the flow-balance equations is as follows:

$$\boldsymbol{\lambda} = (\mathbf{I} - \mathbf{p}^T)^{-1} \mathbf{a} \quad (2)$$

After calculating the arrival rate $\boldsymbol{\lambda}$, each node is analyzed independently as follows. Let P_{nj} indicate the probability that n customers are present at node j . The quantities P_{0j} and P_{nj} are calculated as:

$$P_{0j} = \frac{1}{\sum_{i=0}^{m_j-1} \frac{(\lambda_j / \mu_j)^i}{i!} + \frac{(\lambda_j / \mu_j)^{m_j}}{m_j!} \frac{1}{1 - \lambda_j / (m_j \mu_j)}} \quad (3)$$

$$P_{nj} = \begin{cases} \frac{(\lambda_j / \mu_j)^n}{n!} P_{0j} & \text{if } 0 \leq n < m_j \\ \frac{(\lambda_j / \mu_j)^n}{m_j! m_j^{(n-m_j)}} P_{0j} & \text{if } n \geq m_j \end{cases} \quad (4)$$

Expected queue length at node j (excluding customers being served) is calculated as

$$L_{qj} = \frac{P_{0j}(\lambda_j / \mu_j)^{m_j} \rho_j}{m_j!(1 - \rho_j)^2} \quad (5)$$

where $\rho_j = \lambda_j / (m_j \mu_j)$. The expected queue length at the nodes indicates the number of aircraft in that center which are subjected to delays due to air traffic congestion.

Expected number of customers at the node being served and waiting, is given by

$$L_j = L_{qj} + \frac{\lambda_j}{\mu_j} \quad (6)$$

Expected waiting time in queue excluding time while being served is

$$W_{qj} = \frac{L_{qj}}{\lambda_j} \quad (7)$$

The expected waiting time in the queue indicates the average delay experienced by the aircraft due to congestion.

Expected system time including both waiting and service times is given by:

$$W_j = W_{qj} + \frac{1}{\mu_j} \quad (8)$$

The expected system time indicates the total flight time through a center including the delays due to congestion.

The quantities calculated above can be used to quantify the traffic flow efficiency through a given node as

$$\text{Traffic Flow Efficiency} = \frac{\mathbf{E}(\text{System Time}) - \mathbf{E}(\text{Wait Time})}{\mathbf{E}(\text{System Time})} \quad \text{i.e.} \quad E_j = \frac{\mathbf{E}(W_j) - \mathbf{E}(W_{qj})}{\mathbf{E}(W_j)} \quad (9)$$

where $\mathbf{E}()$ denotes the expected value. The traffic flow efficiency along a path in the airspace can be obtained as

$$E_{\text{path}} = \frac{\sum_{j=1}^n \{\mathbf{E}(W_j) - \mathbf{E}(W_{qj})\}}{\sum_{j=1}^n \mathbf{E}(W_j)} \quad (10)$$

where $j = 1..n$ denotes all the nodes traversed along the path.

VI. Results from the Queuing Model

This section presents the results of the queuing model analysis for a flight route from an airport in the Los Angeles Center (ZLA) to an airport in the New York Center (ZNY). The flight route passes through Los Angeles (ZLA), Denver (ZDV), Minneapolis (ZMP), Cleveland (ZOB), Chicago (ZAU) and New York (ZNY) Centers. Various metrics presented in the previous section such as wait time, system time, efficiency at each node and path efficiency are then calculated. For the sake of brevity only some of the results are presented in the following.

As mentioned earlier in section IV, the number of parallel servers per node can be estimated by analyzing the maximum number of aircraft that can be simultaneously served by the airspace under consideration. For the current center level queuing model, the number of parallel servers per node is obtained by summing the sector capacities of all sectors within a given center. However this leads to an over-estimation of the number of parallel servers available. This is because some of the terminal area sectors may be saturated and may be exhibiting queuing behavior while most of the enroute sectors are operating below capacity. The coarse resolution of the center level model tends to average this effect out resulting in no observed queuing for the center considered as a whole. To demonstrate the kind of analysis that can be performed with this queuing model, the center capacities are reduced by appropriate scaling so that queuing effects are observed. Note that the results presented in the following section are based on these scaled values of the center capacities and do not reflect the current operating condition of the NAS. Future work will develop queuing models at lower resolution such as the sector level model or the latitude-longitude grid model where the queuing phenomenon will be captured accurately and no scaling of the capacities will be required.

Figure 9 shows the variation in the total flight time including delay due to congestion with the departure rate at the airports. Note that all other parameters are held at their nominal values when the departure rate is varied. Red circles indicate nominal values. Each curve in the figure shows two distinct regions. Below a critical value of the external arrival rate, the system time remains constant indicating uncongested traffic. Beyond the critical value, the

system time increases rapidly with the external arrival rate. This is because of the delays due to traffic congestion. Figure 10 shows the variation of the total flight time including delays due to congestion with the service time. Note that the service time is the nominal flight time for a single flight without any interactions with other aircraft. In other words, it is the unimpeded flight time. Below a critical value of the service time, the system time is the same as the service time. Beyond the critical value the system time increases rapidly with increase in service time because of additional delays due to congestion. Figure 11 shows the variation of the total flight time with the center capacity or the maximum number of aircraft that can simultaneously fly within a Center. The system time decreases with the increase in center capacity and two distinct regions of congested and non-congested traffic are seen similar to the earlier figures.

Figure 12 through Figure 14 show the variation of path efficiency with respect to the external arrival rate, service time and center capacity respectively. The observations are analogous to the observations for Figure 9 through Figure 11. Figure 15 shows the variation of the throughput or the mean flow rate at the Center with the external arrival rate. Figure 15 illustrates that the throughput is linearly related to the external arrival rate as indicated in Equation (2).

The above results indicate that the variation in the traffic flow metrics can be obtained from the variation in the queuing model parameters such as the arrival rates, service times, routing probabilities and the center capacities. The uncertainty models that relate the trajectory uncertainties due to aviation operations, precision of navigation and control to the queuing model parameters are being developed. A procedure to relate trajectory uncertainty to one of the queuing model parameter, service time is described in Reference 20. For instance, the variation in the aircraft weight will cause a change in the time taken by the aircraft to climb to cruise altitude, or in terms of the queuing model, the service time for the climb segment. Once the model relating weight variation to the variation in the service time for the climb segment is available, Figure 10 and Figure 13 can be used to relate the weight variation to the total flight time through the system and the system efficiency. As other uncertainty models and queuing network models are developed, the effect of trajectory uncertainties due to aviation operations, precision of navigation and control on the traffic flow efficiency can be studied.

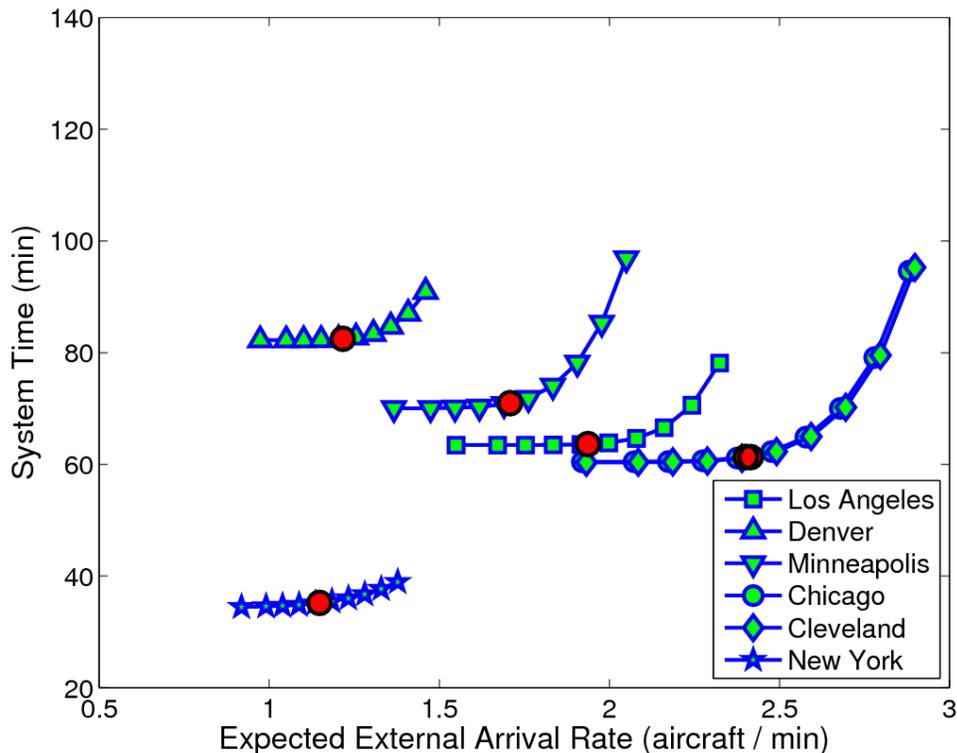


Figure 9. Variation of system time with expected external arrival rate (Aircraft departure rate at airports)

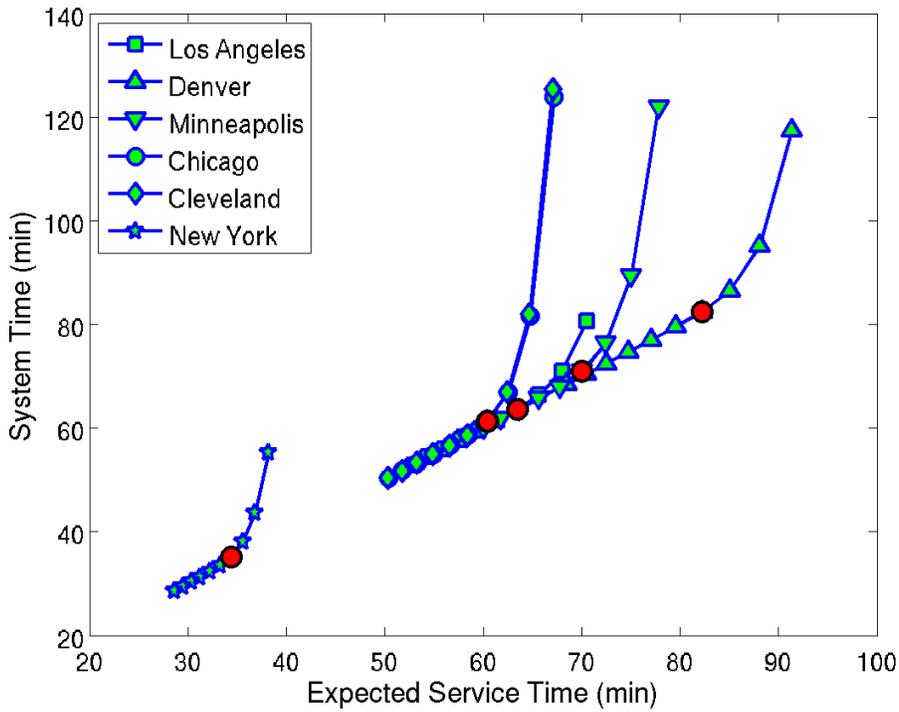


Figure 10. Variation of the system time with expected service time

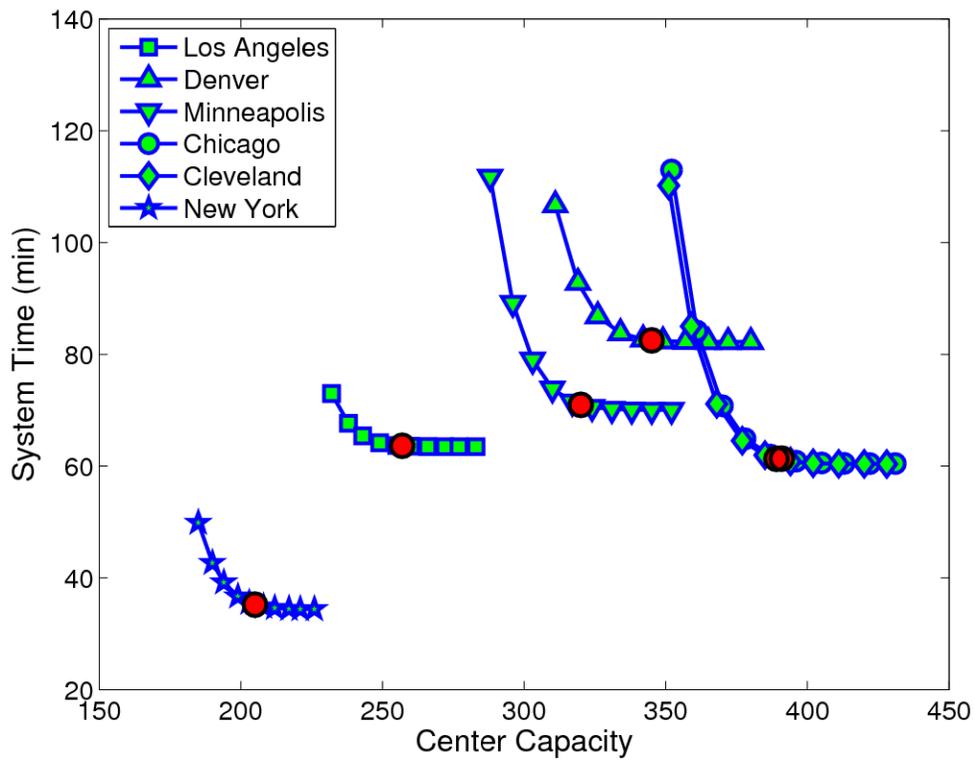


Figure 11. Variation of the system time with the center capacity

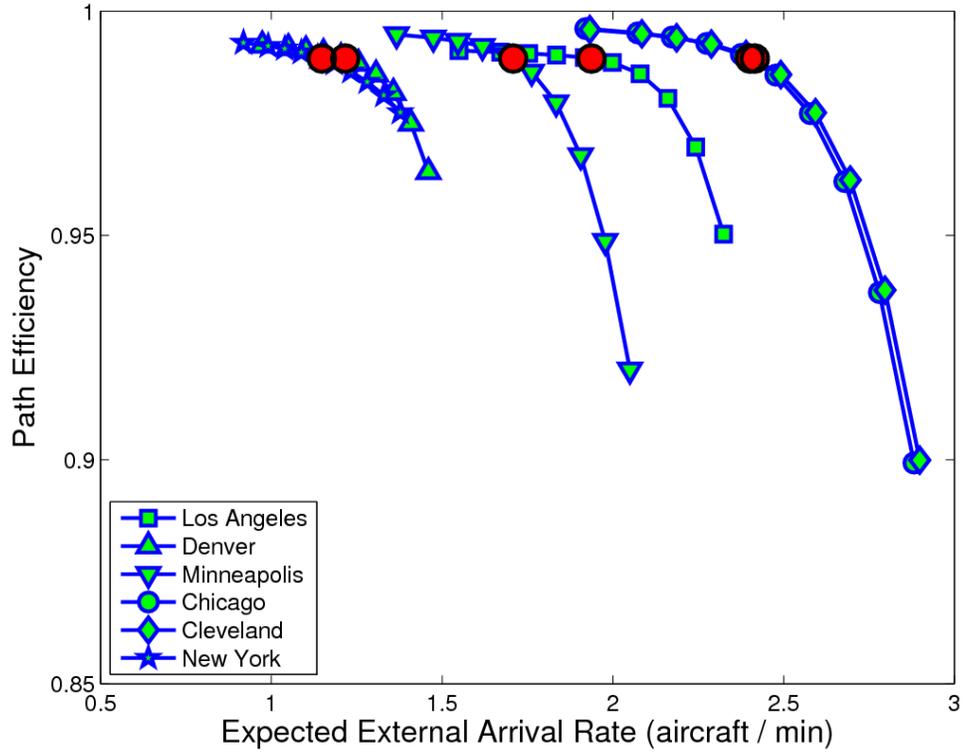


Figure 12. Variation of path efficiency with external arrival rate (Aircraft departure rate at airports)

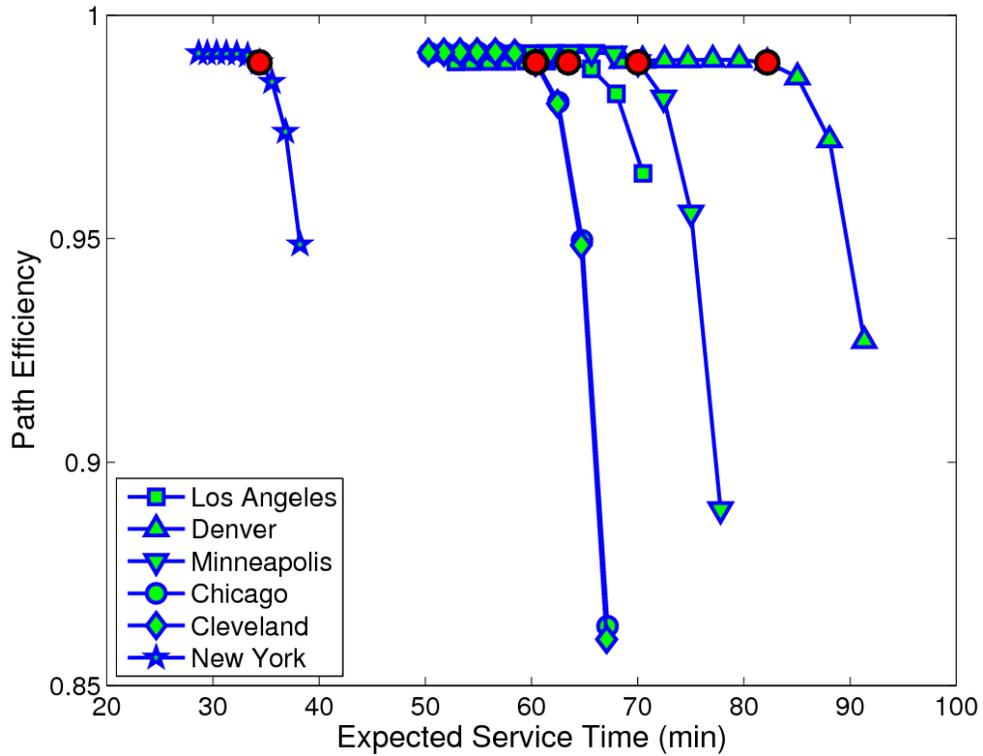


Figure 13. Variation of the path efficiency with expected service time

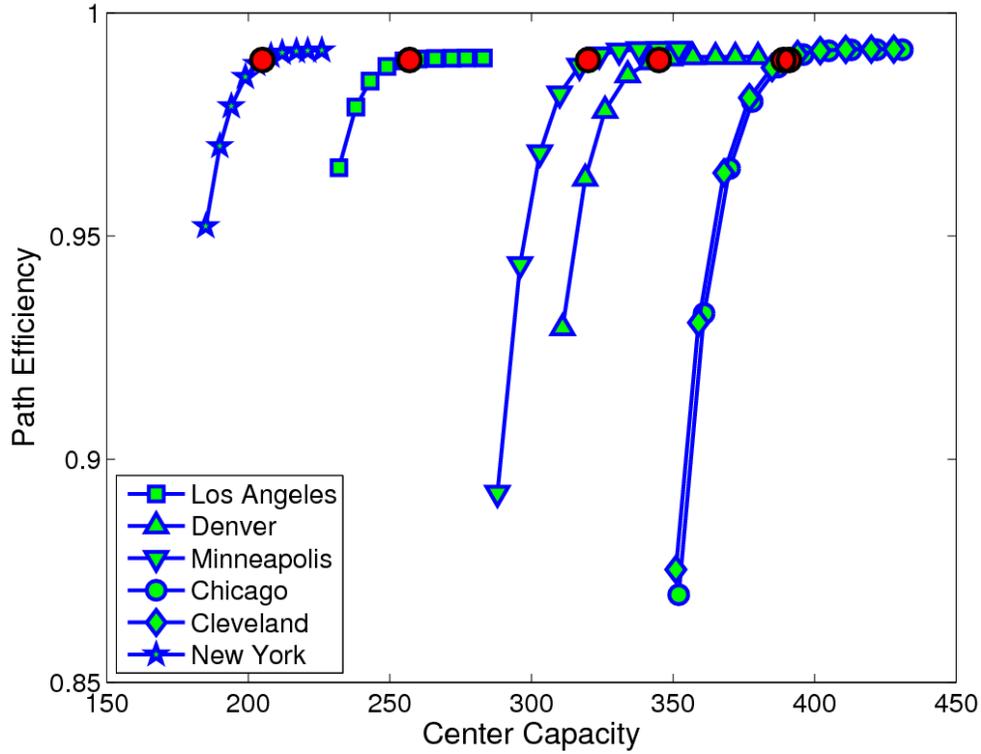


Figure 14. Variation of path efficiency with the center capacity

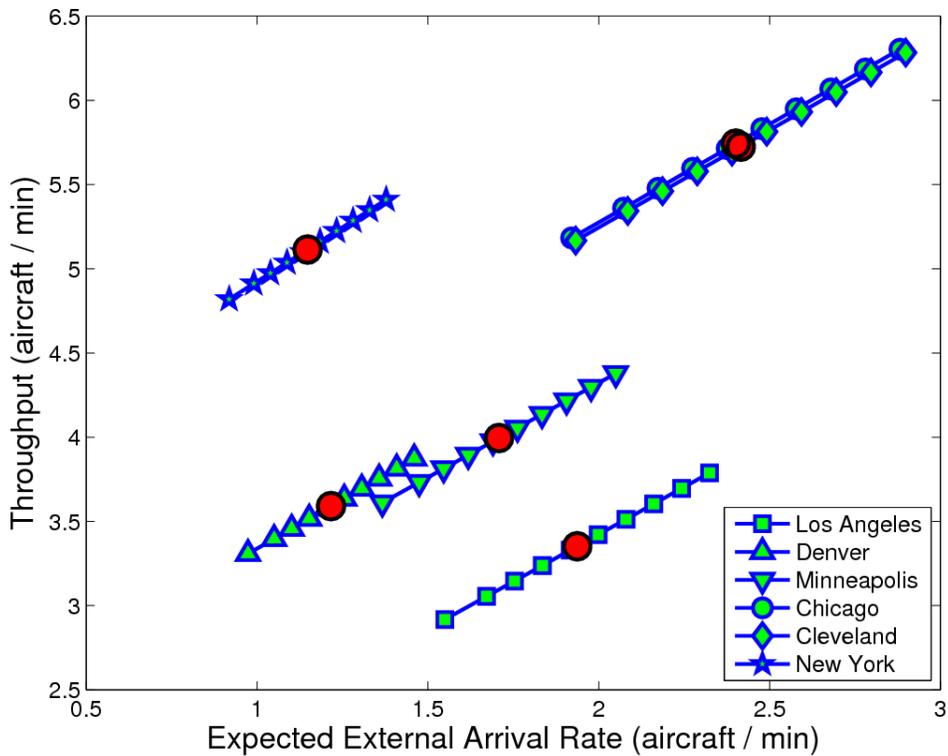


Figure 15. Variation of throughput with expected external arrival rate (Aircraft departure date at airports)

VII. Conclusions and Future Work

This paper presented a methodology to analyze the impact of trajectory uncertainty and precision on air traffic flow efficiency using queuing theory. The central idea is to model every quantifiable uncertainty in aviation operations and cast it into queuing model parameters such as arrival rate distributions, queuing time distributions, transition probabilities and number of servers per node. Queuing network analysis can then help establish the relationships between the system parameters and the metrics indicating traffic flow efficiency. This approach proposes the construction of queuing models of the national airspace system at various spatial and temporal resolutions to perform the analysis. As an example, the present research developed a Center-level open Jackson queuing network model of the NAS with M/M/m nodes and discussed some of the traffic flow efficiency metrics.

The present work assumed exponential distributions for the inter-arrival and service times for mathematical tractability. Future work will address the use of other distributions that more closely match the observed inter-arrival and service times. Also, most of the congestion in the airspace occurs at airports or in the terminal areas while en-route traffic is mostly uncongested. Future research will include models of the airports and the terminal areas to more accurately capture these phenomena.

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References

- ¹ Swenson, H., Barhydt, R., Landis, M., "Next Generation Air Transportation System (NGATS) Air Traffic Management (ATM)-Airspace Project (External Release)", NASA Report, June 2006.
- ² Bilimoria, K. D., Sridhar, B., Chatterji, G. B., Sheth, G., and Grabbe, S., "FACET: Future ATM Concepts Evaluation Tool," *3rd USA/Europe Air Traffic Management R&D Seminar*, Naples, Italy, June 2000.
- ³ Raytheon ATMSDI Team, "Airspace Concept Evaluation System Build 2 Software User Manual," NASA Ames Research Center, Moffett Field, CA, November 2003.
- ⁴ Saaty, T. L., *Elements of Queuing Theory with Applications*, Dover, New York, NY, 1961.
- ⁵ Erlang, A. K., "Solution of Some Probability Problems of Significance for Automatic Telephone Exchanges," *Elektroteknikeren*, Vol. 13, pp. 5-13, 1917.
- ⁶ Pearcey, T., "Delay in Landing of Air Traffic," *Journal of the Royal Aeronautical Society*, Vol. 52, pp. 799-812, 1948.
- ⁷ Bell, G. E., "Operations Research into Air Traffic Control," *Journal of the Royal Aeronautical Society*, Vol. 53, pp. 965-978, 1949.
- ⁸ Bell, G. E., "Queuing Problems in Civil Aviation," *Operations Research Quarterly*, Vol. 3, pp. 9-11, 1952.
- ⁹ Modi, J., "A Nested Queue Model for The Analysis of Air Traffic Control Sectors," *Transportation Research*, pages 219-224, 8 (3), 1974.
- ¹⁰ Polhemus, N., "Construction and Use of Continuous Autoregressive Models for Traffic Indices," *Transportation Research, Part B: Methodological*, pages 271-279, 14 (3), September 1980.
- ¹¹ Frolow, I. and Sinnott, J., "National Airspace System Demand and Capacity Modeling," *Proceedings of the IEEE*, pages 1618-1624, 77 (11), November 1989.
- ¹² Wieland, F., "Limits to Growth: Results From The Detailed Policy Assessment Tool," *AIAA/IEEE Digital Avionics Systems Conference - Proceedings*, pages 9.2-1-9.2-8, 2, 1997.
- ¹³ Long, D., Lee, D., Johnson, J., Gaier, E. and Kostyuk, P., "Modeling Air Traffic Management Technologies with a Queuing Network Model of the National Airspace System," Technical Report *NASA/CR-1999-208988*, Langley Research Center, Hampton Virginia, 2004.
- ¹⁴ M. Callahan and F. Wieland, "Are Air Traffic Models Chaotic?" *AIAA/IEEE Digital Avionics Systems Conference - Proceedings*, pages 7F41-7F48, 2, 2001.
- ¹⁵ Rakas, J. and Schonfeld, P. "Deterministic models for degraded airside capacity and delays," *Journal of Transportation Engineering*, pages 545-554, 130 (5), September/October, 2004
- ¹⁶ Bäuerle, N., Engelhardt-Funke, O. and Kolonko, M., "On the Waiting Time of Arriving Aircrafts and the Capacity of Airports with One or Two Runways," *European Journal of Operational Research*, pages 1180-1196, 177, 2007.
- ¹⁷ Wieland, F., "Investigating the Volume-Delay Relationship at Congested Airports," *Collection of Technical Papers - 6th AIAA Aviation Technology, Integration, and Operations Conference*, pages 450-456, 1, 2006.
- ¹⁸ Cavcar, A. and Cavcar, M., "Impact of Aircraft Performance Characteristics on Air Traffic Delays," *Turkish Journal of Engineering and Environmental Sciences*, pages 13-23, 28 (1), 2004.
- ¹⁹ John F. Shortle, Donald Ross and Briam L. Mark, "Efficient Simulation of the National Airspace System," *Proceedings of the 2003 Winter Simulation Conference*, Vol. 1, pp. 441-448, 7-10 Dec. 2003.

- ²⁰ Kim J., Palaniappan K., Menon P. K., Subbarao K., and Thippavong J., "Trajectory Uncertainty Modeling for Queuing Analysis of the NAS," *The 8th AIAA Aviation Technology, Integration, and Operations (ATIO) Conference*, Anchorage, Alaska, 14 - 19 Sep 2008.
- ²¹ Cassandras, C. G., and LaFortune, S., *Introduction to Discrete Event Systems*, Kluwer, Boston, MA, 1999.
- ²² Hiller, F. S., and Lieberman, G. J., *Operations Research*, Holden-Day, San Francisco, 1974.
- ²³ German, R., *Performance Analysis of Communication Systems: Modeling with Non-Markovian Stochastic Petri Nets*, John Wiley, West Sussex, UK, 2000.
- ²⁴ Ciardo, G. and Li, G., "Efficient Approximate Transient Analysis for a class of Deterministic and Stochastic Petri Nets," *IEEE International Computer Performance and Dependability Symposium*, pages 34-43, Sep 1998.
- ²⁵ A. Bobbio, A. Horvath, M. Scarpa, M. Telek, "Acyclic Discrete Phase Type Distributions: Properties and a Parameter Estimation Algorithm," *Performance Evaluation*, pages 1-32, Vol. 54, No. 1, September 2003.
- ²⁶ W. Bux, U. Herzog, "The Phase Concept: Approximation of Measured Data and Performance Analysis," in *Computer Performance*, pages 23-38, 1977
- ²⁷ M. Malhotra, A. Reibman, "Selecting and Implementing Phase Approximations for Semi-Markov models," *Stochastic Models*, Vol. 9, No. 4, pp.473-506, 1993
- ²⁸ Sridhar, B., Chatterji, G. B. and Grabbe, S., "Benefits of direct-to tool in National Airspace System," *IEEE Transactions on Intelligent Transportation Systems*, Vol. 1, No. 4, Dec. 2000, pp. 190-198.
- ²⁹ Menon, P. K., Sweriduk, G. D., and Bilimoria, K. D., "A New Approach for Modeling, Analysis and Control of Air Traffic Flow," *AIAA Guidance, Navigation, and Control Conference*, August 5-8, 2002, Monterey, CA. Also in the *Journal of Guidance, Control and Dynamics*, Vol. 27, No. 4, 2004, pp. 737-744.
- ³⁰ Menon, P. K., Sweriduk, G. D., Lam, T., Cheng, V. H. L., and Bilimoria, K. D., "Air Traffic Flow Modeling, Analysis and Control," *AIAA Guidance, Navigation and Control Conference*, August 11-14, 2003, Austin, TX.
- ³¹ Menon, P. K., Sweriduk, G. D., Lam, T., and Diaz, G. M., "Methods for Assessing the Stability and Robustness of Distributed ATM Systems," Final report prepared under NASA Contract No. NAS2-02041, Optimal Synthesis Inc, January 23, 2004.
- ³² Menon, P. K., Sweriduk, G. D., Lam, T., Diaz, G. M., and Bilimoria, K. D., "Computer-aided Eulerian Air Traffic Flow Modeling and Predictive Control," *AIAA Guidance, Navigation and Control Conference*, August 16-19, 2004, Providence, RI, Also in the *Journal of Guidance, Control and Dynamics*, Vol. 29, No. 1, January-February 2006, pp. 12-19.
- ³³ Sridhar, B., Soni, T., Sheth, K. S., and Chatterji, G. B., "An Aggregate Flow Model for Air Traffic Management," *AIAA Guidance, Navigation and Control Conference*, August 16-19, 2004, Providence, RI.
- ³⁴ Roy, S., Sridhar, B., and Verghese, G. C., "An Aggregate Dynamic Stochastic Model for Air Traffic Control," *Proceedings of the 5th USA/Europe ATM 2003 R & D Seminar*, Budapest, Hungary, 23-27 June, 2003.
- ³⁵ Grabbe, S., Sridhar, B., "Congestion Management with an Aggregate Flow Model," *AIAA Guidance, Navigation and Control Conference*, August 15-18, 2005, San Francisco, CA.
- ³⁶ Sridhar, B., and Menon, P. K., "Comparison of Linear Dynamic Models for Air Traffic Flow Management," *IFAC World Congress*, Budapest, Hungary, June 2005.
- ³⁷ Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, McGraw-Hill, New York, NY, 1991.